Electric Motors.

Unit - I - Problems.

1) A 250V shunt motor takes a total current of 20A. The shunt field and armature resistances are 200Ω & 0.3Ω respectively. Determine (i) value of back emf (ii) gross mechanical power in the armature.

Given Data.

$V = 250V$, $I_L = 20A$; $R_{sh} = 200\Omega$; $R_a = 0.3\Omega$

(i) Back EMF

$E_b = V - I_a R_a$

Find I_a. $I_L = I_a + I_{sh}$. $I_{sh} = \frac{V}{R_{sh}} = \frac{250}{200} = 1.25A$.

$I_a = I_L - I_{sh} = 20 - 1.25$

$I_a = 18.75A$.

$E_b = V - I_a R_a = 250 - (18.75 \times 0.3)$

$$E_b = 244.4V$$

(ii) Mechanical Power Developed = $P_m = E_b I_a$.

$P_m = 244.4 \times 18.75$

$$P_m = 4582.5 \text{ watts}$$

2) A 230V motor has an armature circuit resistance of 0.6Ω. If the full load armature current is 30A, and no load armature current is 4A. Find the change in back emf from no load to full load.

Given Data

$V = 230\,V$; $R_a = 0.6\Omega$; $I_a = 30A$ (full load); $I_{a(Noload)} = 4A$.

Change in Back EMF from No load to Full load.

At No load

$E_{b(noload)} = V - I_a R_a = 230 - (4 \times 0.6) = 227.6\,V$

$E_{b(full\ load)} = V - I_a R_a = 230 - (30 \times 0.6) = 212\,V$

Change in Back EMF $= 227.6 - 212 = 15.6\,V$.

3) A 4 pole motor is fed at 440V and takes an armature current of 50A. The resistance of the armature circuit is 0.28Ω. The armature winding is wave connected with 888 conductors and useful flux per pole is 0.023wb. Calculate the speed of the motor.

Given Data

$P = 4$; $V = 440V$; $I_a = 50A$; $R_a = 0.28\Omega$; $A = 2$ (wave winding)

$Z = 888$; $\phi = 0.023\,wb$; $N = ?$

$E_b = V - I_a R_a = 440 - (50 \times 0.28) = 426\,V$

$$E_b = \frac{\phi Z N}{60} \frac{P}{A}$$

$$N = \frac{E_b \times 60}{\phi Z} \times \frac{A}{P} = \frac{426 \times 60}{0.023 \times 888} \times \frac{2}{4}$$

$N = 626$ rpm.

4) The counter emf of a shunt motor is 227 V, the field resistance is 160 Ω & field current is 1.5A. If the line current is 39.5 A, Find the armature resistance. Also find the armature current when the motor is stationary.

Given Data.

$E_b = 227V$; $R_{sh} = 160\Omega$; $I_{sh} = 1.5A$; $I_L = 39.5A$.

$R_a = ?$ & I_a when motor is stationary.

$V = I_{sh} R_{sh} = 1.5 \times 160 = 240V.$

$I_a = I_L - I_{sh} = 39.5 - 1.5 = 38A.$

$V = E_b + I_a R_a = 227 + (38 \times R_a)$

(i) $R_a = \frac{V - E_b}{I_a} = \frac{240 - 227}{38} = 0.342\Omega$

(ii) I_a when motor is stationary

$E_b = 0.$ at stationary.

$I_a = \frac{V}{R_a} = \frac{240}{0.342} = 701.5 A$

5). Calculate the value of torque established by the armature of a 4-pole motor having 774 conductors, two paths in parallel, 24 mWb flux per pole when the total armature current is 50A.

Given Data:

$P = 4$; $Z = 774$; $A = 2$; $\phi = 24 mWb$; $I_a = 50A$.

$T_a = ?$

$$T_a = 0.159\ \phi\ z\ I_a \frac{P}{A}$$

$$= 0.159 \times 774 \times 24 \times 10^{-3} \times 50 \times \frac{4}{2}$$

$$\boxed{T_a = 295.35\ Nm}$$

6) An armature of a 6-pole machine, 75cm in diagmetor has 664 conductors each having an effective length of 30cm and carrying a current of 100A. If 70% of total conductors lie simultaneously in the field of averge flux density 0.85 wb/m2, calculate (i) armature torque (ii) horse power output at 250 rpm.

Given Data.

$P = 6$; $d = 75\ cm$; $Z = 664$; $l = 30\ cm \rightarrow 0.3m$; $I_a = 100A$.

$r = 37.5\ cm$

$= 0.0375\ m$. $B = 0.85\ wb/m^2$

Effective conductors $Z = \frac{70}{100} \times 664 = 465$

(i) $F = Bil = 0.85 \times 100 \times 0.3 = 25.5N$.

$T = F \times r = 25.5 \times 0.375 = 9.56\ Nm$.

$T_a = 465 \times 9.56 = \underline{4445.4}\ Nm$.

(ii) HP at 250 rpm.

$$P_o = \frac{2\pi N T_a}{60 \times 746} \text{ in HP} = \frac{2\pi \times 250 \times 4445.4}{60 \times 746}$$

($T_a \rightarrow T_{sh}$)

$$\boxed{P_o \text{ in HP} = 156\ HP}$$

7) A 230V dc shunt motor takes a current of 40A and runs at 1100 rpm. If armature and shunt field resistances are 0.25Ω & 230Ω respectively, find the torque developed by the armature.

Given Data

$V = 230V$; $I_L = 40A$, $N = 1100$ rpm; $R_a = 0.25\Omega$,

$R_{sh} = 230\Omega$. $T_a = ?$

$$T_a = \frac{9.55 E_b I_a}{N} = 9.55 \frac{(V - I_a R_a)}{N}$$

$$I_a = I_L - I_{sh} = 40 - \frac{230}{230} \qquad I_{sh} = \frac{V}{R_{sh}} = \frac{230}{230} = 1A.$$

$$I_a = 40 - 1 = 39A.$$

$$T_a = 9.55 \frac{(230 - (39 \times 0.25))}{1100} = 74.66 \text{ NM}$$

8). A 230V dc shunt motor takes 5A at no load and runs at 1000 rpm. Calculate the speed when loaded and taking a current of 30A. The armature and field resistances are 0.2Ω & 230Ω respectively.

Given Data.

$V = 230V$; $I_{L1} = 5A$; $N_1 = 1000$ rpm.

$I_{L2} = 30A$; $N_2 = ?$ $R_a = 0.2\Omega$ $R_{sh} = 230\Omega$.

No load

$$E_{b1} = V - I_{a1} R_a$$

$$I_{a1} = I_{L1} - I_{sh} = 5 - \frac{230}{230} = 5 - 1 = 4A.$$

$$E_{b1} = 230 - (4 \times 0.2) = 229.2V$$

At Load.

$E_{b2} = V - I_{a2}R_a.$

$I_{a2} = I_{L2} - I_{sh} = 30 - 1 = 29$

$E_{b2} = 230 - 29 \times 0.2 = \underline{224.2\ V}$

Shunt motor ; ϕ constant.

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

$$N_2 = \frac{E_{b2}}{E_{b1}} \times N_1 = \frac{224.2}{229.2} \times 1000 = \underline{978\ rpm}$$

9) A 6-pole lap wound shunt motor has 500 conductors The armature and shunt field resistances are 0.05 Ω & 25 Ω respectively. Find the speed of motor, if it takes 120A from dc supply of 100V. Flux per pole is 20mwb.

Given Data

$P = 6$; $A = P$; $Z = 500$; $R_a = 0.05\ \Omega$; $R_{sh} = 25\ \Omega$.

$I_L = 120A$; $V = 100V$; $\phi = 20\ m\ wb$; $N = ?$

$E_b = V - I_a R_a$; $I_a = I_L - I_{sh}$; $I_{sh} = \frac{V}{R_{sh}} = \frac{100}{25}$

$I_{sh} = 4A.$

$I_a = 120 - 4 = 116A.$

$E_b = 100 - (116 \times 0.05). = 94.2\ V.$

$$E_b = \frac{\phi Z N}{60} \frac{P}{A}.$$

$$N = \frac{E_b \times 60 \times A}{\phi Z P} = \frac{94.2 \times 60 \times 6}{20 \times 10^{-3} \times 500 \times 6} = \underline{565\ rpm}$$

10) A 200V series motor takes a current of 100A and runs at 1000 rpm. The total resistance of the motor is 0.1Ω & the field is unsaturated. Calculate

(i) Percentage change in torque and speed if the load is changed that motor current is 50A.

(ii) the motor current and speed if the torque is halved.

Given Data.

$V = 200V$; $I_L = 100A$ (= I_a) ; $N_1 = 1000 rpm$;

Total resistance $(R_a + R_{se}) = 0.1Ω$; field is unsaturated.

(i) % change in T & N if $I_L = 50A$.

when $I_L = 100A$

$I_{sh} = V$

$I_{L1} = 100A$; $I_{a1} = I_{L1} = 100A$

$I_{a2} = 50A$.

Field is unsaturated $\phi \propto I_a$.

$$\boxed{T_a \propto I_a^2}$$

$$\frac{T_{a2}}{T_{a1}} = \frac{I_{a2}^2}{I_{a1}^2}$$

$$T_{a2} = \frac{I_{a2}^2}{I_{a1}^2} \times T_{a1} = \frac{50^2}{100^2} \times T_{a1} = 0.25\, T_{a1}$$

$$\%\ \text{Change in Torque} = \frac{T_{a1} - T_{a2}}{T_{a1}} \times 100 = \frac{T_{a1} - 0.25\, T_{a1}}{T_{a1}} \times 100$$

$$= 75\%$$

Speed N_2

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_a}$$

$E_{b1} = V - I_{a1} R_a = 200 - (100 \times 0.1) = 190V$

$E_{b2} = V - I_{a2} R_a = 200 - (50 \times 0.1) = 195V$

$$N_2 = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \times N_1 = \frac{195}{190} \times \frac{100}{50} \times 1000$$

$$\boxed{N_2 = 2052 \text{ rpm.}}$$

(ii) I_a & N if the torque is halved.

$\underline{I_a}$

$T_a \propto I_a^2$

$T_{a1} \propto I_{a1}^2$ & $T_{a2} \propto I_{a2}^2$.

$I_{a1} = 100A$

~~$I_{a2} = 50A$.~~

$T_{a2} = \frac{T_{a1}}{2}$.

$E_{b1} = 190V$

~~$E_{b2} = 195V$.~~

$$\frac{I_{a2}^2}{I_{a1}^2} = \frac{T_{a2}}{T_{a1}} \Rightarrow I_{a2}^2 = \frac{T_{a2}}{T_{a1}} \times I_{a1}^2$$

$$I_{a2}^2 = \frac{0.5 \cancel{T_{a1}}}{\cancel{T_{a1}}} \times 100^2$$

$$\boxed{I_{a2} = 70.7A.}$$

Speed if Torque halved.

$E_{b2} = V - I_{a2} R_a = 200 - (70.7 \times 0.1) = \underline{192.93}V$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}$$

$$N_2 = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}} \times N_1 = \frac{192.93}{190} \times \frac{100}{70.7} \times 1000$$

$$\boxed{N_2 = 1436 \text{ rpm.}}$$

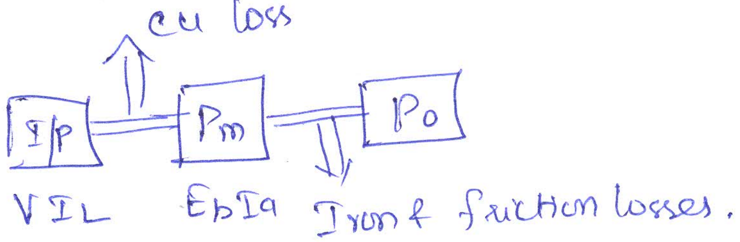
(iii) A 220V shunt motor takes a total current of 80A and runs at 800 rpm. Shunt field resistance and armature resistance are 50Ω & 0.1Ω respectively. If iron and friction losses amount to 1600W. Find (i) copper losses (ii) armature torque (iii) shaft torque (iv) efficiency.

Given Data.

$V = 220V$; $I_L = 80A$; $N = 800$ rpm ; $R_{sh} = 50Ω$; $R_a = 0.1Ω$

Iron & Friction losses = 1600W.

(i) Copper loss.



$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{50} = 4.4A.$$

$$I_a = I_L - I_{sh} = 80 - 4.4 = 75.6A.$$

$$E_b = V - I_a R_a = 220 - (75.6 \times 0.1) = 212.44V$$

Input power $P_{in} = V I_L = 220 \times 80 = 17600W.$

Power Developed in the armature $= E_b I_a$.

$$P_m = 212.44 \times 75.6 = 16050W$$

$P_{in} = P_m + $ cu loss.

Cu losses $= P_{in} - P_m = 17600 - 16050 = 1550W$

(ii) Armature Torque

$$T_a = 9.55 \frac{E_b I_a}{N} = 9.55 \times \frac{212.44 \times 75.6}{800}$$

$$\boxed{T_a = 192 \text{ Nm}}$$

(iii) Shaft Torque.

$P_m = P_o +$ Iron & Mechanical loss.

$P_o = P_m -$ Iron & mech. loss.

$= 16050 - 1600$

$$\boxed{P_o = 14450 \text{ W.}}$$

Shaft Torque $= 9.55 \times \frac{P_o}{N}$

$$P_o = \frac{2\pi N T_{sh}}{60} \Rightarrow T_{sh} = \frac{60}{2\pi} \frac{P_o}{N}$$

$$= 9.55 \frac{P_o}{N}.$$

$$T_{sh} = 9.55 \times \frac{14450}{800}$$

$$\boxed{T_{sh} = 172.5 \text{ Nm}}$$

(iv) Efficiency. η.

$$\eta = \frac{P_{out}}{P_{in}} = \frac{14450}{17600} \times 100 = \underline{82.1\%}$$

12) A 220V dc shunt motor having an armature resistance of 0.25Ω carries an armature current of 50A and runs at 600rpm. If the flux is reduced by 10% by field regulator, find the speed assuming load torque remains the same.

Given Data.

$V = 220V$, $R_a = 0.25\Omega$; $I_a = 50A$; $N = 600 rpm$

$\phi_2 \Rightarrow$ reduced 10%. $\phi_2 = 90\% \cdot \phi_1 = 0.9\phi_1$

$$\frac{\phi_2}{\phi_1} = 0.9$$

Load Torque remains same.

$T \propto I_a \phi$ $\quad \frac{T_{a2}}{T_{a1}} = \frac{I_{a2}}{I_{a1}} \times \frac{\phi_2}{\phi_1}$

$$1 = \frac{I_{a2}}{I_{a1}} \times \frac{\phi_2}{\phi_1}$$

$$\frac{I_{a2}}{I_{a1}} = \frac{\phi_1}{\phi_2} \Rightarrow I_{a2} = \frac{\phi_1}{\phi_2} \times I_{a1} = \frac{1}{0.9} \times 50$$

$$\boxed{I_{a2} = 55.6A}$$

$$E_{b2} = V - I_{a2} \times R_a = 220 - (55.6 \times 0.25)$$

$$E_{b2} = 206.1V.$$

$$E_{b1} = V - I_{a1} R_a = 220 - (50 \times 0.25)$$

$$\boxed{E_{b1} = 207.5V}$$

$$\frac{N2}{N1} = \frac{Eb2}{Eb1} \times \frac{\phi_1}{\phi_2}$$

$$\frac{N2}{600} = \frac{206.1}{207.5} \times \frac{1}{0.9}$$

$$\boxed{N2 = 662 \text{ rpm.}}$$

(13) A shunt motor supplied at 230V runs at 900 rpm while taking armature current of 30A; the resistance of armature circuit being 0.4Ω. Calculate the resistance required in series with the armature circuit to reduce the speed to 500 rpm, assuming that the armature current is 25A.

Given Data.

$V = 230V$; $N_1 = 900$ rpm ; $Ia_1 = 30A$; $Ra_1 = 0.4\Omega$

$N2 = 500$ rpm ; $Ia2 = 25A$; $Rc = ?$ $Rt = Ra + Rc$

$Eb1 = V - Ia_1 Ra = 230 - (30 \times 0.4) = 218V.$

$Eb2 = V - Ia2\ Rt = 230 - (25\ Rt)$

$\phi_1 = \phi_2$ (shunt motor)

$$\frac{Eb2}{Eb1} = \frac{N2}{N1}$$

$$Eb2 = \frac{N2}{N1} \times Eb1 = \frac{500}{900} \times 218 = 121.11V.$$

$E_{b2} = V - I_{a2} R_t$

$121.11 = 230 - 25 \times R_t$.

$25 R_t = ~~351.11~~ \; 108.89$

$R_t = \frac{108.89}{25} = 4.356\Omega$.

$R_t = R_c + R_a$

$4.356 = R_c + 0.4$

$$\boxed{R_c = 3.956\Omega.}$$

(14) A 220V dc series motor runs at 900 rpm when taking a line current of 40A. The armature resistance and series field resistance are 0.06Ω & 0.04Ω respectively. If current taken remains same, calculate series resistance required to reduce the speed to 600 rpm.

<u>DC Series Motor.</u>

$V = 220V$; $N_1 = 900rpm$; $I_{a1} = I_{L1} = 40A$.

$R_a = 0.06\Omega$; $R_{se} = 0.04\Omega$. $I_{a2} = I_{a1}$

$R_c = ?$ $N_2 = 600$.

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \cdot \frac{I_{a1}}{I_{a2}} = \frac{E_{b2}}{E_{b1}} = \frac{V - I_{a2}(R_a + R_{se} + R_c)}{V - I_{a1}(R_a + R_{se})}.$$

$$\frac{600}{900} \Rightarrow \frac{220 - 40(0.06 + 0.04 + R_c)}{220 - 40(0.06 + 0.04)}$$

$$\frac{600}{900} = \frac{220 - 4 - 40 R_c}{216}$$

$$144 = 216 - 40 R_c$$

$$40 R_c = 72$$

$$\boxed{R_c = 1.8\,\Omega}$$

15) A 60kW, 500V DC shunt motor has a lap connected armature with 400 conductors. Flux/pole is 0.05 wb and full load efficiency is 90%. Its armature resistance is 0.1Ω & shunt field resistance is 250Ω. Find speed for full load.

Given Data:

$P_o = 60kW$; $V = 500V$; lap $P = A$; $Z = 400$;

$\phi = 0.05$ wb; $\eta = 90\%$; $R_a = 0.1\Omega$; $R_{sh} = 250\Omega$.

Output Power $= 60 \times 10^3$ watts.

$$\text{Efficiency } \eta = \frac{O/P \text{ Power}}{I/P \text{ Power}}$$

$$0.9 = \frac{60 \times 10^3}{P_i}$$

$$\text{Input Power} = \frac{60 \times 10^3}{0.9} = 66666.67 \text{ watts.}$$

$$P_i = V I_L$$

$$66666.67 = 500 I_L$$

$$\boxed{I_L = 133.33 A}$$

$I_a = I_L - I_{sh}$. $I_{sh} = \frac{V}{R_{sh}} = \frac{500}{250} = 2A$.

$= 133.33 - 2$.

$$\boxed{I_a = 131.33A.}$$

Find Torque.

$T_a = 0.159 \times \phi Z I_a \times P/A \quad (P = A)$.

$= 0.159 \times 0.05 \times 400 \times 131.33$

$$\boxed{T_a \Rightarrow 417.64 \text{ Nm}}$$

Find Eb

$E_b = V - I_a R_a = 500 - (131.33 \times 0.1)$

$$\boxed{E_b = 486.867 \text{ V}}$$

Find N.

$T_a = 9.55 \frac{E_b I_a}{N}$

$N = 9.55 \frac{E_b I_a}{T_a} = 9.55 \times \frac{486.867 \times 131.33}{417.64}$

$$\boxed{N = 1462.1 \text{ rpm.}}$$

16) A 500V, 37.3KW, 1000rpm dc shunt motor has on full load an efficiency of 90%. The armature circuit resistance is 0.24Ω & there is total voltage drop of 2V at the brushes. The field current is 1.8A. Determine (i) full load line current (ii) full load shaft torque (iii) Total resistance in motor starter to limit the starting current to 1.5 times the full load current (iv) Back Emf.

Given Data

$V = 500V$; $P_o = 37.3KW$; $N = 1000$ rpm;

$\eta = 90\%$; $R_a = 0.24\Omega$; Brush Drop = 2V

$I_{sh} = 1.8A$;

(i) Full load line current.

$$\eta = \frac{P_o}{P_i} \Rightarrow 0.9 = \frac{37.3 \times 10^3}{P_i}$$

$$P_i = \frac{37.3 \times 10^3}{0.9} = 41444.4 \text{ watts}.$$

$$P_i = V I_L$$

$$41444.4 = 500 \times I_L$$

$$I_L = 82.88 A.$$

(ii) Shaft Torque.

$$P_o = \frac{2\pi N T_{sh}}{60} \Rightarrow T_{sh} = \frac{P_o \times 60}{2\pi N} = \frac{37.3 \times 10^3 \times 60}{2 \times \pi \times 1000}$$

~~$T_{sh} = 5.94$ Nm~~ $T_{sh} = 356.4$ Nm

(iii) Back EMF

$V = E_b + I_a R_a + BCD$ → Brush contact drop.

$E_b = V - I_a R_a - BCD$.

$= 500 - (81.08 \times 0.24) - 2$.

$$\boxed{E_b = 478.54\ V}$$

$I_a = I_L - I_{sh}$

$= 82.88 - 1.8$

$= 81.08\ A$

(iv) R in starter to limit 1.5 times full load current.

1.5 times full load current / Starting current $= 1.5 \times 82.89$.

$I_L(start) = 124.335\ A$

$I_a(start) = I_L(start) - I_{sh}$.

$= 124.335 - 1.8$

$I_a(start) = 122.535\ A$

$E_b = V - I_a(R_a + R_{start}) - BCD$.

E_b at starting $= 0$.

$V = E_b + I_a(R_a + R_{start}) + BCD$

$V = I_a(R_a + R_{start}) + BCD$

$500 = 122.535(0.24 + R_{start}) + 2$.

$$\boxed{R_{start} = 3.824\ \Omega}$$